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Adaptive Actuator/Component Fault Compensation for Nonlinear Systems

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An active fault compensation control law is developed for a class of nonlinear systems to guarantee closed-loop stability in the presence of faults, based on a neural network representation of the fault dynamics. Changes in the systems due to faults are modeled as unknown nonlinear functions. The closed-loop stability of the robust fault compensation scheme is established in the Lyapunov's sense. The nonlinear fault function is represented by a neural network, then an adaptive corrective control law is formulated to ensure system stability. The main contributions presented are the design of the fault compensation and corrective control law of nonlinear systems with unmatched uncertainties and the stability analysis of the closed-loop systems in the presence of fault modeling errors. Applications of the proposed design indicate that the fault compensation control law is effective for a nonlinear fermentation process. © 2008 American Institute of Chemical Engineers *AIChE J,* 54: 2404–2412, 2008
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Introduction

In the past decades, people are faced with more complex systems when the performance requirements increase. Actuator, sensor or component faults drastically change the complex system behavior. Therefore, it is necessary to improve reliability of a system by diagnosing faults of individual components, and applying fault-tolerant control (FTC) systems.[1–10]

Based on the nature of the design, fault-tolerant control can be categorized into passive or active types. Passive fault-tolerant control uses the same control scheme before and after the fault, without specific accommodating parameters, typically by introducing a conservative control law.[11–20] For active fault-tolerant control, control reconfiguration takes place, following the diagnosis of a fault to counteract any dynamic changes caused by the fault, typically by a reconfiguration of the feedback control law. An excellent overview of active FTC has been given by Zhang and Jiang.[21] Different methods for dealing with the reconfiguration problem have been reported. Most of them adopt the following methods: neural networks,[33] output feedback,[34] fuzzy logic systems,[7] adaptive control,[22,23] eigenstructure assignment,[24] Markov model,[32] multiple-model tracking,[25,26] and compensation via additive input design.[27–29] In particular, an excellent overview of the fault accommodation has been discussed in Patton (1997).[30] Typical sensor and actuator faults are concerned and respective accommodation strategies are designed. For example, sensor fault accommodation for MIMO systems has been discussed in Tortora et al.,[8] sensor performance can be monitored using self-validating devices, and fault accommodation strategies are then derived with the

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intent of trading-off optimality for computational simplicity and transparency. A compensation method for actuator faults with known plant dynamics has been formulated in Boskovic et al.[31] and a nonlinear fault accommodation controller has been designed by making use of redundancy.[9]

The objective of this article is to propose and analyze a robust fault compensation scheme for nonlinear systems. Changes in the systems due to faults are modeled as an unknown nonlinear function. The closed-loop stability of the robust fault compensation scheme is established in the Lyapunov's sense, i.e., the nonlinear fault functions are represented by a neural network, then an adaptive corrective control law is formulated to ensure system stability.

The remainder of the article is organized as follows. The robust fault detection and fault adaptation scheme is introduced, followed by a fault compensation scheme and relevant proofs. Then a fermentation process example is given to demonstrate the effectiveness of the proposed methods. Finally, conclusions are given.

Robust Fault Detection and Fault Adaptation

Consider the following nonlinear systems

$$\begin{aligned} \dot{x} &= a(x) + \Delta\xi(x) + b(x)u + \beta(t-T)f(x) \\ y &= c(x) \end{aligned} \tag{1}$$

where $x \in R^n$, $u \in R^m$ are the state and input vector of the systems, respectively, $\Delta\xi(x)$ is the unmatched model uncertainties, $f(x)$ characterizes the changes in the dynamics due to a fault. The nonlinear fault function f is multiplied by a switching function $\beta(t - T)$

$$\beta(t-T) = diag\,(\beta_1(t-T), \beta_2(t-T), \ldots, \beta_n(t-T))$$

where

$$\beta_i(t-T) = \begin{cases} 0 & \text{if } t < T \\ 1 & \text{if } t \geq T \end{cases} \qquad i = 1, 2, \ldots, n$$

and T is the fault occurrence time. The normal ("healthy") systems, in the absence of any fault, is described by

$$\begin{aligned} \dot{x} &= a(x) + \Delta\xi(x) + b(x)u \\ y &= c(x) \end{aligned} \tag{2}$$

Remark 1: Faults are described by the vector $f(x)$, assumed to be zero before the fault occurrence, and non-zero after the fault occurrence. In this article, only actuator and component faults estimation and compensation are considered. In fact, $f(x)$ in systems (1) can be time-varying, it can also represent component faults. Sensor faults are not considered since specific analysis can be done in, e.g., Tortora et al.[8]

We now propose a robust fault diagnosis scheme, which consists of a nonlinear state estimator and a learning algorithm. We consider a nonlinear adaptive state estimator as follows

$$\dot{\hat{x}} = -\Lambda(\hat{x} - x) + a(x) + b(x)u + \hat{f}(x, \hat{\vartheta}) \tag{3}$$

where $\hat{x} \in R^n$ is the estimated state vector, and $\hat{f}(x, \hat{\vartheta})$ represents an online approximation model with $\hat{f}(x, \hat{\vartheta}_0) = 0$. $\Lambda = diag(\lambda_1, \lambda_2, \ldots, \lambda_n)$ with $-\lambda_i < 0$ is the ith pole.

The next step in the construction of the fault diagnosis scheme is the design of the learning algorithm for updating the weights $\hat{\vartheta}$ of the online approximator. According to the Lyapunov synthesis method, the adaptation of the weights is chosen such that the derivative of a specified Lyapunov function has some desirable stability properties. Let $e = x - \hat{x}$, we have the following relation after the fault occurs

$$\dot{e} = \Lambda e + \Delta\xi(x) + f(x) - \hat{f}(x, \hat{\vartheta})$$

Note that

(1) $\hat{f}(x, \hat{\vartheta})$ is to approximate $f(x)$ when it occurs.

(2) Before fault $f(x)$ occurs, $f(x) = 0$, $\hat{f}$ is set to zero. Hence $e(t)$ is bounded because Λ is stable and $\Delta\xi(x)$ is bounded. This bound, denoted as $e^0(t)$, is the normal bound for fault detection.

(3) When $e(t)$ exceeds its normal bound, adaption of $\hat{f}$ is activated with the following learning scheme:[1]

$$\dot{\hat{\vartheta}} = P\{\Gamma Z^T D[e]\},\ \hat{\vartheta}(0) = \hat{\vartheta}_0 \tag{4}$$

where $\Gamma = \Gamma^T$ is the positive-definite learning rate matrix, and $Z = \partial\hat{f}(x, \hat{\vartheta})/\partial\hat{\vartheta}$. The dead zone operator

$$D[\bullet] = \begin{cases} 0, & \text{if } \max_i\{|e_i| - e_i^0(t)\} < 0 \\ e & \text{otherwise} \end{cases},$$

where assuming $|\Delta\xi_i(x)| \leq \xi_i(x)$, the normal bound

$$e_i^0(t) = \int_0^t e^{-\lambda_i(t-\tau)}\xi_i(x(\tau))d\tau,\ i = 1, 2, \ldots, n$$

which can be easily implemented as the output of a linear filter, whose input is given by the bound $\xi_i(x(t))$. Note that, as long as ξ_i is bounded, the output of the stable filter remains bounded as well. The transfer function is $1/(s + \lambda_i)$. When λ_i increases, $e_i^0(t)$ decreases. For a fermentation process, a slow process, false alarm will frequently occur if λ_i is very large. Smaller alarm delay is not very important for a slow process. So the time varying dead zone threshold $e_i^0(t)$ needs to be sufficiently large to prevent false alarms. We found, $\lambda_i \in [-5, -1]$ is available for the fermentation process. The projection operator P (which restricts the parameter estimate vector $\hat{\vartheta}$ to a compact, convex region $M_{\hat{\vartheta}}$) is used to avoid parameter drift, a phenomenon that may occur with standard adaptive laws in the presence of modeling uncertainties.[1,37,38] If $M_{\hat{\vartheta}}$ is chosen to be a hypersphere of radius $\overline{M}$, then the aforementioned adaptive law can be expressed as

$$\dot{\hat{\vartheta}} = \Gamma Z^T D[e] - \chi^*\Gamma(\hat{\vartheta}\hat{\vartheta}^T/(\hat{\vartheta}^T\Gamma\hat{\vartheta}))\Gamma Z^T D[e],\ \hat{\vartheta}(0) = \hat{\vartheta}_0 \tag{5}$$

where χ^* denotes the indicator function given by

$$\chi^* = \begin{cases} 0 & \text{if } \{|\vartheta| < \overline{M} \text{ or } (|\vartheta| = \overline{M} \text{ and } \hat{\vartheta}\Gamma Z^T e \leq 0)\} \\ 1 & \text{if } \{|\vartheta| = \overline{M} \text{ and } \hat{\vartheta}\Gamma Z^T e > 0\} \end{cases}$$

The presence of model error (denoted by $\Delta\xi(x)$ in the state equation) causes a nonzero state estimation error $e(t)$, even in the absence of a fault. The dead zone operator stops adaptation of the approximator when the modulus of the estimation error $e(t)$ is below its corresponding threshold. The fault detection time T_d is defined as follows

$$T_d = \inf \bigcup_{i=1}^{n} \{t \geq T : |e_i| > e_i^0(t)\} \tag{6}$$

In the absence of any faults and with the initial weights of the online approximator, it can be easily verified that the state estimation satisfies

$$\begin{aligned} |e_i(t)| &= \left| \int_0^t e^{-\lambda_i(t-\tau)} \Delta\xi_i(x(\tau)) d\tau \right| \\ &\leq \int_0^t e^{-\lambda_i(t-\tau)} \xi_i(x(\tau)) d\tau = e_i^0(t) \end{aligned} \tag{7}$$

Therefore, the robustness of the detection scheme, i.e., the ability to avoid any false alarm in the presence of modeling uncertainty, is guaranteed.[1]

The conservation of the detection threshold is as follows:

A large threshold can lead to detection delays and subsequent degradation in the system's overall performance. A smaller value for the threshold is lead to false alarms.

The main contributions of this paper, shown in the next section, are the design of the corrective control law for nonlinear systems with unmatched uncertainties, and the stability analysis of the closed-loop systems in the presence of the fault modeling errors.

Fault Compensation of Nonlinear Systems

For convenience, we denote $\beta(t - T) f(x)$ of systems (1) as $f(x)$. And the following assumptions are used in the design process of controller.

Assumption 1: There exists $u = u^a(y)$ and a C^1 function $v(x) : R^n \mapsto R^+$, such that

$$k_1|x|^2 \leq v(x) \leq k_2|x|^2 \tag{8}$$

$$\begin{aligned} \frac{\partial v(x)}{\partial x}\left(a(x) + b(x)u^a(y)\right) &\leq -k_3 \left|\frac{\partial v(x)}{\partial x}\right|^2 \\ &\leq -k_4 v(x) \end{aligned} \tag{9}$$

where k_1, k_2, k_3 and k_4 are positive constants. Note that $u^a(y)$ is the nominal feedback control law without unmatched uncertainties.

Assumption 2: For systems (1), there exist the nonsingular function matrix $G(\bullet) \in R^{n\times m}, g(\bullet), \zeta(\bullet) \in R$ such that

$$\frac{\partial v(x)}{\partial x} b(x) = (G(y)y)^T \tag{10}$$

$$\left|\frac{\partial v(x)}{\partial x}\right| \leq g(x)\zeta(y) \tag{11}$$

where v is defined in Assumption 1.

Assumption 3:

$$|\Delta\xi(x)| \leq \varphi(x)\phi(y) \tag{12}$$

Remark 2: Consider the uncertainty of systems (1), from the inequality (12), we specify the bounded uncertainties of the systems. Assumptions 2 and 3 are basic requirements for (1) about the problem of output feedback stabilization, as in Dawson et al.[35] and Yan et al.[36]

For systems (1), we use a neural network to learn fault function $f(x)$. If x is the input vector to the neural network, the input and output representation is $f(x) = WS(x)$ where W is a $n \times L$ weight matrix, and $S(x)$ is a vector with $S_i(x)$, $i = 1,2,\ldots,L$. Here, $S_i(x) = \prod_{j\in J_i}[s(x_j)]^{l_j(i)}$ with J_i, $i = 1,2,\ldots,L$ and $l_j(i)$ is nonnegative integers. From the theorem 2.1 of the reference,[37] it can be shown that there exists an optimized matrix W^*, such that $|f(x) - W^*S(x)| \leq \varepsilon$ is satisfied for any given $\varepsilon > 0$. In other words, $W^*S(x)$ can approximate $f(x)$ to any degree of accuracy. With the previous representation, systems (1) can be rewritten as

$$\dot{x} = a(x) + \Delta\xi(x) + b(x)u + W^*S(x) + \varepsilon(x) \tag{13}$$

where, $\varepsilon(x)$ is equal to the difference between $f(x)$ and $W^*S(x)$. Hence, $|\varepsilon(x)| = |f(x) - W^*S(x)| \leq \varepsilon$ is the estimation error.

If we denote W as the estimate of the unknown but optimal weight matrix W^*, then

$$\dot{x} = a(x) + \Delta\xi(x) + b(x)u - \tilde{W}S(x) + WS(x) + \varepsilon(x)$$

where $\tilde{W} = W - W^*$ and it has the appropriate dimension.

We design an adaptive fault compensation controller based on the normal control law u_N. The main controller structure is to design a compensation control law u_C to be added to the normal law u_N in order to attenuate the effect of the dynamics caused by the faults on the systems.

Remark 3: The controller design procedure is as follows: design a control u_N for the normal systems, and an additional control u_C for fault compensation, $u = u_N + u_C$ as the new control after the occurrence of a fault.

Theorem 1: Under Assumptions 1-3, we can design a controller in the form of the following

$$u = u_N + u_C \tag{14}$$

$$u_N = u^a(y) + u^b(y) \tag{15}$$

where u^a is given in Assumption 1, and let

$$u^b = -\frac{\zeta^2(y)\phi^2(y)}{2\delta^2}(G(y)^T)^+ y \tag{16}$$

$$u_C = \frac{b^T(x)WS(x)}{\lambda[1 + \|b(x)\|^2]} + \frac{b^T(x)\Theta}{\bar{\lambda}[1 + \|b(x)\|^2]} \tag{17}$$

where $\Theta \in R^n$ and $\Theta = [\theta,0,\ldots,0]^T$. Then, the state x is ultimately consistently bounded by the set

$$D = \left\{x \in R^n : v(x) \leq \frac{\mu_1}{\rho_1\alpha}, \frac{\rho_3}{\rho_2} \leq \rho_1 \leq 1\right\} \tag{18}$$

with the following adaptive weight update law

$$\dot{W} = -\beta W + 2\rho_1 \frac{\partial v}{\partial x} S^T(x) \tag{19}$$

$$\dot{\theta} = -\gamma_1 \theta + \rho_1 \frac{\partial v}{\partial x} \tag{20}$$

where

$$\delta \leq \frac{|g^{-1}(x)||\varphi^{-1}(x)|}{\left(k_3\left|\frac{\partial V_1(x)}{\partial x}\right|\right)^{-1}}.$$

The parameters of $\lambda, \overline{\lambda}, \rho_1, \rho_2, \rho_3, \alpha$ and μ_1 are determined as in the proof.

The proof of the aforementioned theorem is divided into the following two steps: Step 1, we prove that there exists a nominal controller $u_N = u^a + u^b$ and a Lyapunov function $V_1(x)$ for the "healthy" systems described by $\dot{x} = a(x) + \Delta\xi(x) + b(x)u$, such that the closed-loop of the normal systems is stable; and 2, we prove that the state x is ultimately consistently bounded, using the control law stated in the theorem.

Proof: Step 1. Stability of the fault-free systems

Substituting the controller Eqs. of 14 and 15 into systems (1), we have

$$\dot{x} = a(x) + \Delta\xi(x) + b(x)(u^a + u^b) \tag{21}$$

Defining a positive function

$$V_1(x) = v(x) \tag{22}$$

we have

$$\dot{V}_1(x) = \frac{\partial V_1}{\partial x}(a(x) + b(x)u^a) + \frac{\partial V_1}{\partial x}(\Delta\xi(x) + b(x)u^b) \tag{23}$$

From Assumption 1, we have

$$\frac{\partial V_1(x)}{\partial x}\left(a(x) + b(x)u^a(y)\right) \leq -k_3\left|\frac{\partial V_1(x)}{\partial x}\right|^2 \tag{24}$$

From Assumptions 2 and 3 and the structure of $u^b(x)$, we have

$$\begin{aligned}
&\frac{\partial V_1(x)}{\partial x}(\Delta\xi(x) + b(x)u^b) \\
&= \frac{\partial V_1(x)}{\partial x}\Delta\xi(x) + (G(y)y)^T u^b \\
&= \frac{\partial V_1(x)}{\partial x}\Delta\xi(x) - (G(y)y)^T \frac{\zeta^2(y)\phi^2(y)}{2\delta^2}(G(y)^T)^+ y \\
&\leq g(x)\varphi(x)\zeta(y)\phi(y) - \frac{\zeta^2(y)\phi^2(y)}{2\delta^2}|y|^2 \\
&\leq \frac{\delta^2}{2}g^2(x)\varphi^2(x) + \frac{1}{2\delta^2}\zeta^2(y)\phi^2(y) - \frac{1}{2\delta^2}\zeta^2(y)\phi^2(y) \\
&= \frac{\delta^2}{2}g^2(x)\varphi^2(x)
\end{aligned} \tag{25}$$

since

$$\delta \leq \frac{|g^{-1}(x)||\varphi^{-1}(x)|}{\left(k_3\left|\frac{\partial V_1(x)}{\partial x}\right|\right)^{-1}}$$

and (25), we have

$$\frac{\partial V_1(x)}{\partial x}(\Delta\xi(x) + b(x)u^b) \leq \frac{1}{2}k_3\left|\frac{\partial V_1(x)}{\partial x}\right|^2 \tag{26}$$

Thus, we obtain

$$\dot{V}_1(x) \leq -\frac{1}{2}k_3\left|\frac{\partial V_1(x)}{\partial x}\right|^2 \tag{27}$$

From (27), the stability of the "healthy" systems is proven.

Proof: Step 2. Stability of the faulty systems

Define a Lyapunov function for systems (1) of the following form

$$V_2(x, \tilde{W}, \tilde{\theta}) = \rho_1 V_1(x) + \frac{1}{2}tr\{\tilde{W}^T\tilde{W}\} + \frac{1}{2}\tilde{\theta}^2 \tag{28}$$

with $\tilde{\theta} = \theta - \varepsilon$, then the derivative of V_2 is

$$\begin{aligned}
\dot{V}_2 = {}&\rho_1\left(\frac{\partial V_1}{\partial x}(a(x) + b(x)(u^a + u^b) + \Delta\xi(x))\right) \\
&+ \rho_1\frac{\partial V_1}{\partial x}b(x)u_C - \rho_1\frac{\partial V_1}{\partial x}\tilde{W}S(x) \\
&+ \rho_1\frac{\partial V_1}{\partial x}WS(x) + \rho_1\frac{\partial V_1}{\partial x}\varepsilon(x) + tr\{\dot{W}^T\tilde{W}\} + \tilde{\theta}\dot{\theta}
\end{aligned} \tag{29}$$

Using Eq. 23, we obtain

$$\begin{aligned}
\dot{V}_2 = {}&\rho_1\dot{V}_1 + \rho_1\frac{\partial V_1}{\partial x}b(x)u_C + \rho_1\frac{\partial V_1}{\partial x}WS(x) \\
&+ \rho_1\frac{\partial V_1}{\partial x}\varepsilon(x) + tr\{\dot{W}^T\tilde{W}\} + \tilde{\theta}\dot{\theta}
\end{aligned} \tag{30}$$

Since $tr\{W^T\tilde{W}\} = \frac{1}{2}\|W\|^2 + \frac{1}{2}\|\tilde{W}\|^2 - \frac{1}{2}\|W^*\|^2$, by substituting $u_C(\lambda,\lambda_1)$ into (30), from Assumption 1, as $\frac{\|b(x)\|^2}{1+\|b(x)\|^2} \leq 1$, let $k_3 = \rho_2 + \rho_3 + \rho_4$, Choosing $\lambda \geq \frac{\rho_1 s}{\sqrt{2\rho_3\beta - s\rho_1}}$, $\bar{\lambda} \geq \frac{\rho_1}{\sqrt{2\rho_1\rho_3\gamma_1 - \rho_1}}$, $\beta > \frac{s^2\rho_1^2}{2\rho_3}$, $\gamma_1 > \frac{\rho_1}{2\rho_3}$ Eq. 30 is transformed into

$$\begin{aligned}
\dot{V}_2 \leq {}&-\rho_1\rho_2\left|\frac{\partial V_1}{\partial x}\right|^2 - \rho_1\rho_3\left|\frac{\partial V_1}{\partial x}\right|^2 - \rho_1\rho_4\left|\frac{\partial V_1}{\partial x}\right|^2 \\
&+ \rho_1 V_1\left|\frac{\partial V_1}{\partial x}\right|\|W\|\left(1 + \frac{1}{\lambda}\right) + \rho_1\left|\frac{\partial V_1}{\partial x}\right|\theta\left(1 + \frac{1}{\bar{\lambda}}\right) \\
&-\frac{\gamma_1}{2}\tilde{\theta}^2 - \frac{\gamma_1}{2}\theta^2 + \frac{\gamma_1}{2}\varepsilon^2 - \frac{\beta}{2}tr\{\tilde{W}^T\tilde{W}\} \\
&+ \frac{\beta}{2}tr\{\tilde{W}^T\tilde{W}\} - \frac{\beta}{2}\|W\|^2 + \frac{\beta}{2}\|W^*\|^2
\end{aligned} \tag{31}$$

If $\frac{\rho_3}{\rho_2} \leq \rho_1 \leq 1$ is satisfied, then Eq. 31 can be changed into

$$\begin{aligned}
\dot{V}_2 \leq {}&-\rho_1\rho_4\left|\frac{\partial V_1}{\partial x}\right|^2 - \frac{\beta}{2}\|\tilde{W}\|^2 + \frac{\beta}{2}\|W^*\|^2 \\
&-\frac{\gamma_1}{2}\tilde{\theta}^2 + \varepsilon^2
\end{aligned} \tag{32}$$

By using Eqs. 9, 32 can be transformed into the following form

$$\dot{V}_2 \le -\frac{\rho_1\rho_4 k_4}{k_3} V_1(x) - \frac{\beta}{2}\|\tilde{W}\|^2 + \frac{\beta}{2}\|W^*\|^2 - \frac{\gamma_1}{2}\tilde{\theta}^2 + \frac{\gamma_1}{2}\varepsilon^2 \quad (33)$$

therefore

$$\dot{V}_2 \le -\alpha V_2 + \mu \quad (34)$$

with $\alpha = \min\left\{\frac{\rho_4 k_4}{k_3}, \beta, \gamma_1\right\}$, $\mu_1 = \frac{\beta}{2}\|W^*\|^2 + \frac{\gamma_1}{2}\varepsilon^2$. Integrating both sides of Eq. 34 yields

$$V_2(t) \le \frac{\mu_1}{\alpha} + \left[V_2(0) - \frac{\mu_1}{\alpha}\right]e^{-\alpha t}, \forall t \ge 0 \quad (35)$$

Due to Eq. 35, it can be deduced that x, W, θ are bounded consistently. From Eq. 28, we have

$$\rho_1 v(x) \le V_2 \quad (36)$$

Therefore

$$v(x) \le \frac{\mu_1}{\rho_1\alpha} + \frac{1}{\rho_1}\left[V_2(0) - \frac{\mu_1}{\alpha}\right]e^{-\alpha t}, \forall t \ge 0 \quad (37)$$

we complete the proof that x is ultimately consistently bounded by the set D.

Remark 4: From the proof of stabilization of the "healthy" systems, we know that the controller of the "healthy" systems defined by u_N is an output feedback composite controller, i.e., $u_N = u^a(y) + u^b(y)$. Compared with the state feedback stabilization of nonlinear systems, the existing results of proposed output feedback stabilization are much less.

Remark 5: According to Eq. 37, we know that both the "healthy" systems (2) by using u_N and the overall systems (1) by adding u_C are stabilized. This guarantees that the nominal systems is stable, with respect to $\Delta\xi(x)$ and the "healthy" systems remains stable once a fault occurs.

Simulation Examples

Simple example

Consider the following systems

$$\dot{x} = \begin{bmatrix} x_1 + x_1^2 \\ x_2 + x_2^2 \end{bmatrix} + \Delta\xi + \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \beta(t-T)f(x)$$
$$y_i = 2x_i, i = 1,2$$

where

$$\Delta\xi(x) = \begin{bmatrix} \theta_1 x_1 x_2 e^{x_2} \\ 2x_2^2 e^{x_2}\sin\theta_2 \end{bmatrix}, \theta_1 \in (-2,2)$$

and $\theta_2,\theta_3 \in (-1, 1)$ are the uncertainty parameters. Thus, $\xi(x) = 2\|x\|^2 e^{\|x\|}$.

We use a three-layer radial basis function network with 10 neurons in the hidden layer and two neurons in the output layer for detecting actuator faults. The 10 centers are distributed uniformly in region $[-1,1]$. The width of each RBF is 0.06. We do not need data to train the neural network offline; it is to be trained online when a fault occurs. However, it needs initial parameters. The initial parameter vector of the network is chosen such that the output of the neural network is zero in D. This can be simply achieved by setting the output weights of the neural network to zero. We set the learning rate as $\Gamma = 2.8I_{2\times 2}$. The size of the hypersphere for the projection algorithm is selected as $\overline{M} = 10$ in the experiment. Set $\lambda_1 = -2, \lambda_2 = -3$. We perform three set of simulations. In the first simulation, we consider a fault that occurs at $t = 4s$. Similar to reference,[1,39] the class of faults is described as follows

$$f_1(x) = \begin{cases} 0 & t \le 4s \\ 1 & 4s < t \le 4.5s \\ 1.5 & 4.5s < t \le 5s \\ 1 & 5s < t \le 5.5s \\ 0 & 5.5s < t \le 6s \end{cases}$$

$$f_2(x) = \begin{cases} 0 & t \le 4s \\ -(0.2 + 0.4(\sin(2\pi x_3/5) + \cos(2\pi x_3/3))) & t > 4s \end{cases}$$

We take $v(x) = 0.5x^T x = 0.5|x|^2$, $u^a = (\, u_1^a \quad u_2^a \,)^T$ with

$$u_1^a = -2 - y_1,$$
$$u_2^a = -2 - y_2$$

and

$$u^b = \frac{|y|^3 e^{2|y|}(y^T)^+ y}{0.02}$$

When f_1 is introduced at $t = 4s$, as shown in Figure 1, we have

$$u_C = \frac{b^T(x)WS(x)}{0.006} + \frac{b^T(x)\begin{bmatrix}\theta \\ 0\end{bmatrix}}{0.006}$$

the weight adaptive law

$$\dot{W} = 3k_0 x^T S^T(x)$$
$$\dot{\theta} = -0.0037\theta + k_0|x|$$

and the set $D = \left\{x \in R^n : v(x) \le \frac{1.8}{k_0}, 0.6 \le k_0 \le 1\right\}$. We choose $k_0 = 0.7$, the faults are introduced at $T = 4s$, the control results are shown in Figure 3.

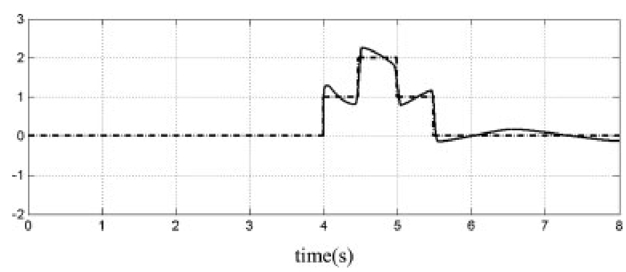

Figure 1. Fault (f_1) and estimation (Dashed: actual. Solid: estimated).

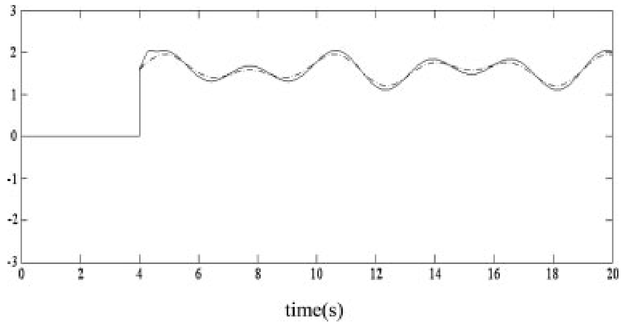

Figure 2. Fault (f_2) and estimation (Dashed: actual. Solid: estimated).

When f_2 is introduced at $t = 4s$, as shown in Figure 2, we have

$$u^F = \frac{b^T(x)WS(x)}{0.005} + \frac{b^T(x)\begin{bmatrix}\theta \\ 0\end{bmatrix}}{0.005}$$

the weight adaptive law

$$\dot{W} = 2k_0 x^T S^T(x)$$
$$\dot{\theta} = -0.00390 + k_0|x|$$

and the set $D = \left\{x \in R^n : v(x) \leq \frac{2.6}{k_0}, 0.8 \leq k_0 \leq 1\right\}$. We choose $k_0 = 0.9$, the faults are introduced at $T = 4s$, the control results are shown in Figure 4.

Figures 3 and 4 (the outer figures) depict the control responses of the two states without any fault. Obviously, the states converge at about $T = 2s$. The two states have the significant changes when the faults are introduced at $T = 4s$, as shown in Figures 3 and 4 (the inner figures). The results of using the proposed compensation control law show that all states converge in spite of the faults (Figures 3 and 4 (the inner figures)). This suggests that the proposed control is effective.

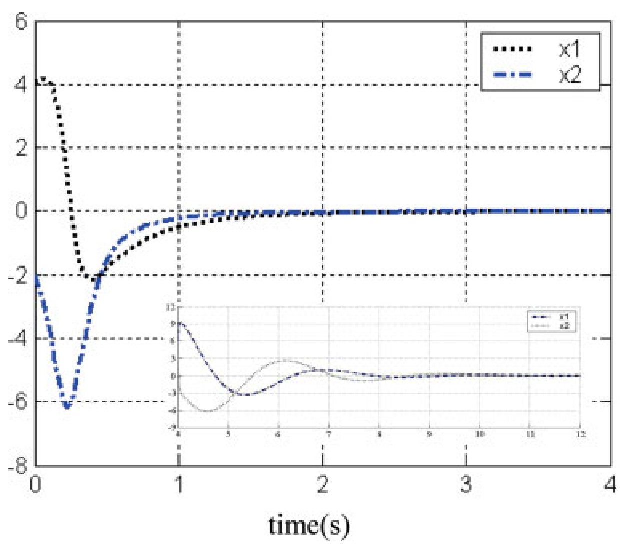

Figure 3. Control responses of the states without any fault (the outer figure) and control responses of the states with fault compensation for fault f_1 (the inner figure).

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

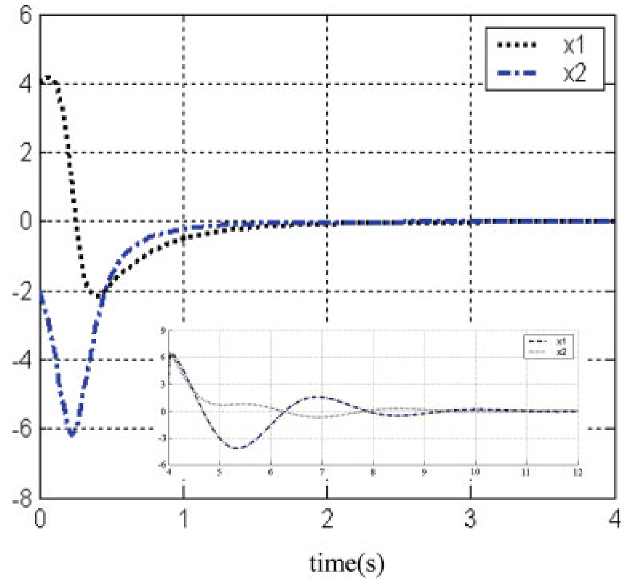

Figure 4. Control responses of the states without any fault (the outer figure) and control responses of the states with fault compensation for fault f_2 (the inner figure).

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

Fermentation process

This section takes a biological fermentation process as a nonlinear process example to show that the control design can result in a stable closed-loop systems in which the states converge in the presence of faults.

The fermentation process is assumed to operate at a constant volume V, with the dynamics of biomass X, substrate S, and toxin concentration C_t, described by the follows

$$\frac{dX}{dt} = \mu X - DX \quad (38)$$

$$\frac{dS}{dt} = -DS - \mu\frac{X}{y_s} \quad (39)$$

$$\frac{dC_t}{dt} = qX^{1/3} - DC_t \quad (40)$$

where the dilution rate D, and the yield coefficient y_s are given by

$$D = \frac{F}{V}, y_s = \frac{y_a\mu}{My_a + \mu}$$

and the nonlinear inhibited specific growth rate is $\mu = \mu_m\left[\frac{S}{K_s+S+S^3/K_i}\right]\left[\frac{K_t}{K_t+C_t^2}\right]$.

Table 1. Fermentation Model Parameters

Volume	V	200[l]
Constant	y_a	0.417
Constant	M	0.0196
Toxin production constant	q	0.0296[l/h(g/l)$^{2/3}$
Maximum specific growth rate	μ_m	0.0135[l/h]
Monod constant	K_s	0.05[g/l]
Substrate inhibition constant	K_i	2150[l^2/g^2]
Toxin inhibition constant	K_t	5.5[g^2/l^2]

The parameters of y, q, μ_m, K_s, K_i, K_t, M are given in Table 1 for the process.

Defining the state as $x = [X\ S\ C_t]^T$, and the input $u = F/V$, Eq. 38–40 become

$$\begin{bmatrix} \frac{dX}{dt} \\ \frac{dS}{dt} \\ \frac{dC_t}{dt} \end{bmatrix} = \begin{bmatrix} \mu X \\ -(M + \mu/y_a)X \\ qX^{1/3} \end{bmatrix} + \begin{bmatrix} -X \\ -S \\ -C_t \end{bmatrix} u$$
$$y_i = x_i$$

and

$$a(x) = \begin{bmatrix} 0.5x_1 \\ -1.4x_1 \\ 0.6x_1^{1/3} \end{bmatrix},\ b(x) = \begin{bmatrix} -x_1 \\ -x_2 \\ -x_3 \end{bmatrix}$$

where $x = col(x_1, x_2, x_3) = [X\ S\ C_t]^T$.

We use a three-layer RBF neural network with 15 neurons in the hidden layer, and three neurons in the output layer for detecting actuator faults. The 15 centers are distributed uniformly in region [−1,1]. The initial parameter vector of the network is chosen, such that the output of the neural network is zero in D. This can be simply achieved by setting the output weights of the neural network to zero. We set the learning rate as $\Gamma = 2.5I_{3\times 3}$. The size of the hypersphere for the projection algorithm is selected as $\overline{M} = 10$ in the experiment. Set $\lambda_1 = -1$, $\lambda_2 = -3$, $\lambda_3 = -5$ since $\lambda_i \in [-5, -1]$ is available for the fermentation process. We perform three set of simulations. In the first simulation, we consider a fault witch occur at $t = 0.5s$.

The modeling uncertainty is assumed to arise out of a 5% inaccuracy in the value of μ. It is also assumed that the uncertainty in μ is at most 10%,[1] i.e.

$$\Delta\xi = \begin{bmatrix} 0.05\mu x_1 \\ 0.12\mu x_1 \\ 0 \end{bmatrix}$$

The bounding function is clearly bounded in any compact region of the state space.

We take $v(x) = 0.5x^T x = 0.5|x|^2$, $u^a = (\,u_1^a\,u_2^a\,u_3^a)^T$ with

$$u^a = \begin{bmatrix} -2.57 - y_1 \\ -0.86 - y_1 \\ -0.34 - 1.25y_1^{2/3} \end{bmatrix}$$

and

$$u^b = \frac{y_1^2 (y^T)^+ y}{0.05}$$

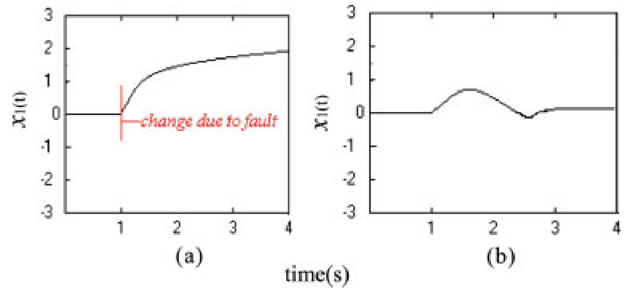

Figure 5. Control response of state $x_1(t)$ (a) without fault compensation, and (b) with fault compensation.

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

The fault is assumed to manifest itself as a nonlinear change (in the system dynamics) described by

$$f_3(x) = \begin{cases} 0 & t \le 1s \\ 0.4\ \sin x_3 & t > 1s \end{cases}$$

$$f_4(x) = \begin{cases} 0 & t \le 0.5s \\ -0.4b(x)u_1 & t > 0.5s \end{cases}$$

When f_3 is introduced at $t = 1s$, we have

$$u^F = \frac{b^T(x)WS(x)}{0.002} + \frac{b^T(x)\begin{bmatrix} \theta \\ 0 \end{bmatrix}}{0.002}$$

the weight adaptive law

$$\dot{W} = 1.29k_0 x^T S^T(x)$$
$$\dot{\theta} = -0.057\theta + k_0|x|$$

and the set $D = \left\{x \in R^n : v(x) \le \frac{0.83}{k_0}, 0.52 \le k_0 \le 1.14\right\}$. We choose $k_0 = 0.8$, the faults are introduced at $T = 1s$, the control results are shown in Figures 5–7. Figures 5a, 6a and 7a depict the control responses of the three states without using of the proposed accommodation strategy. Obviously, the states diverge from the set-point after the occurrence of

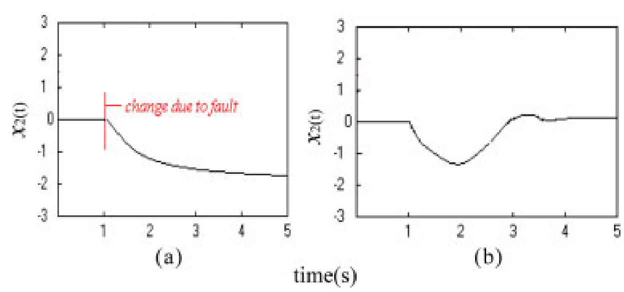

Figure 6. Control response of state $x_2(t)$ (a) without fault compensation, and (b) with fault compensation.

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

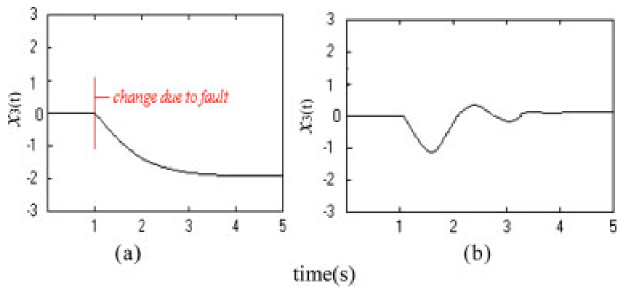

Figure 7. Control response of state $x_3(t)$ (a) without fault compensation and (b) with fault compensation.

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

the faults at $T = 1s$. On the contrary, the results of using the proposed compensation control law show that all states converge despite of the faults, as shown in Figures 5b, 6b and 7b. This suggests that the proposed control is effective.

For the actuator fault f_4 in the fermentation process, when the loss of actuator effectiveness is 40%, and is introduced at $t = 0.5s$, we have

$$u^F = \frac{b^T(x)WS(x)}{0.002} + \frac{b^T(x)\begin{bmatrix}\theta \\ 0\end{bmatrix}}{0.002}$$

the weight adaptive law

$$\dot{W} = 1.5k_0 x^T S^T(x)$$
$$\dot{\theta} = -0.097\theta + k_0|x|$$

The manipulated inputs behave with and without fault compensation are shown in Figure 9. The set $D = \left\{x \in R^n : v(x) \le \frac{0.53}{k_0}, 0.62 \le k_0 \le 1.27\right\}$. We choose $k_0 =$

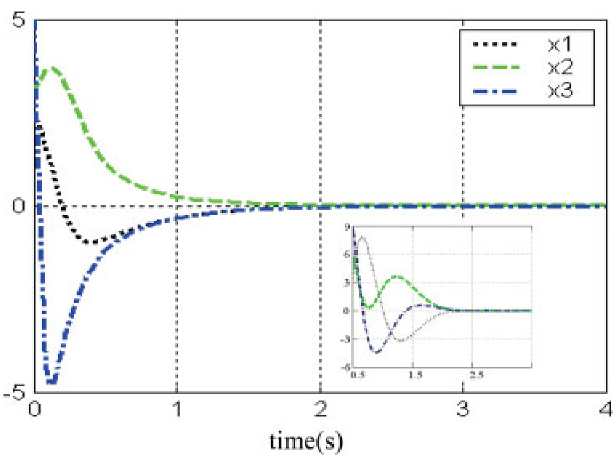

Figure 8. Control responses of the states without any fault (the outer figure) and control responses of the states with fault compensation for fault f_2 (the inner figure).

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

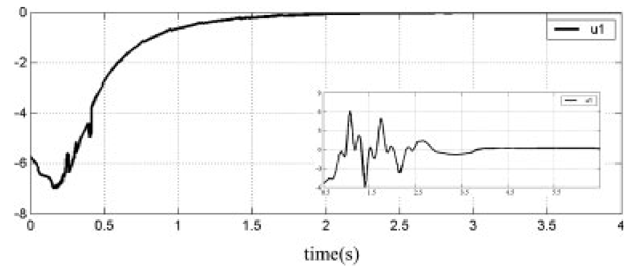

Figure 9. Controller of 'healthy' system and fault compensation controller.

0.9, the control results are shown in Figure 8. (the outer figure) depicts the control responses of the three states without any fault. Obviously, the states converge at about $T = 1.5s$. The three states have the significant changes when the faults are introduced at $T = 0.5s$, as shown in Fig. 8 (the inner figures). The results of using the proposed compensation control law show that all states converge in spite of the faults (Figure 8 (the inner figures)). This suggests that the proposed control is effective. The control law of "healthy" systems and fault compensation control law are shown in Figure 9 (the outer figure and the inner figure).

Conclusions

An active fault compensation control law is developed to ensure the closed-loop stability for a class of nonlinear systems with unmatched uncertainties, using a neural network learning approach. First, the nonlinear fault function is represented by neural network, then an adaptive corrective control law is formulated to ensure system stability. The resulting closed-loop systems are stable based on the corrective control law. The proposed fault-tolerant control design has been shown to be effective for a simple example and a fermentation process.

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